Compressive Sensing of Multichannel Electrocardiogram Signals in Wireless Telehealth System*

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Due to the capacity of compressing and recovering signal with low energy consumption, compressive sensing (CS) has drawn considerable attention in wireless telemonitoring of electrocardiogram (ECG) signals. However, most existing CS methods are designed for reconstructing single channel signal, and hence difficult to reconstruct multichannel ECG signals. In this paper, a spatio-temporal sparse model-based algorithm is proposed for the reconstruction of multichannel ECG signals by not only exploiting the temporal correlation in each individual channel signal, but also the spatial correlation among signals from different channels. In addition, a dictionary learning (DL) approach is developed to enhance the performance of the proposed reconstruction algorithm by using the sparsity of ECG signals in some transformed domain. The approach determines a dictionary by learning local dictionaries for each channel and merging them to form a global dictionary. Extensive simulations were performed to validate the proposed algorithms. Simulation results show that the proposed reconstruction algorithm has a better performance in recovering multichannel ECG signals as compared to the benchmarking methods. Moreover, the reconstruction performance of the algorithm can be further improved by using a dictionary matrix, which is obtained from the proposed DL algorithm.

Keywords: Compressive sensing; multichannel electrocardiogram; signal reconstruction; spatio-temporal sparse; dictionary learning.

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1. Introduction

Electrocardiogram (ECG) signals have been widely used in wireless telehealth systems for diagnosis of heart diseases during the past decade. Reducing energy consumption is a major design concern of wireless telehealth systems since most of such systems are powered by a battery with limited capacity. In general, the most efficient way to achieve energy savings is by compressing the data in the system without losing information quality before data transmission. Compressive sensing (CS) has been attracting considerable attention in wireless telemonitoring of ECG signals as an emerging data compression technique in recent years. It utilizes a measurement matrix to compress a signal and then recovers it from a small number of nonadaptive linear measurements that preserve the structure of the signal. Signal needs to be reconstructed for further data analysis and medical diagnosis in telehealth systems. The state-of-the-art signal reconstruction algorithms for CS are mainly based on $l_1$-norm minimization, $l_0$-norm minimization, $l_p$-norm minimization, greedy iterative, and Bayesian learning. In Ref. 11, the authors presented an algorithm for obtaining sparse solutions of underdetermined systems of linear equations by minimizing the $l_1$-norm using linear programming (LP) techniques. Contrary to the method proposed in Ref. 11, the schemes presented in Ref. 12 and 13 attempt to directly minimize $l_0$-norm, which in general have a 2–3 orders of magnitude faster speed and are able to achieve the same or better accuracy. A new algorithm is proposed to reconstruct signal by minimizing an unconstrained regularized $l_p$-norm with $p < 1$ in the null space of the measurement matrix. It has been shown that using $l_p$ minimization with $p < 1$ can reduce the number of measurements as compared to that with $p = 1$. The studies describe several recovery algorithms where the computation is performed recursively and iteratively, including orthogonal matching pursuit (OMP) and compressive sampling matching pursuit (CoSaMP). In Ref. 17, a Bayesian formalism is employed to estimate the original signal from CS measurements. These approaches presented in the above works are able to reconstruct ECG signals, however, none of them considers the correlation of signals.

Recently, Zhang et al. point out that the block sparse Bayesian learning bound optimization (BSBL-BO) algorithm can be used for CS of ECG signals. The BSBL has a better performance in signal reconstruction as compared to the methods discussed above due to the exploitation of temporal correlation of a signal. In Refs. 20 and 21, the authors presented $l_p^d$ and $l_p^{2d}$-RLS algorithm, and applied them to the reconstruction of ECG signals. The algorithms also exploit temporal correlation, but they do so by promoting sparsity on the first-order and second-order differences of the ECG signals, respectively. However, these methods are only designed for recovering single channel signal. When the number of channels is large, they have to recover the signals channel by channel, in other words, they cannot reconstruct the
multichannel signals at the same time, which is time consuming and therefore not suitable for telemonitoring of multichannel ECG signals in real time. Moreover, none of them considers the correlation among signals of different channels. Thus, in this paper, we exploit both the temporal correlation and inter-channel correlation to reconstruct multichannel signals.

It has been proven that the design of dictionary matrix is crucial to guarantee good reconstruction performance. Considerable research effort has been devoted to the dictionary learning. DL techniques such as the method of optimal directions (MOD), the $K$-mean singular value decomposition (KSVD) algorithm, and online DL (ODL) have received great attention in recent years. Both MOD and KSVD are offline iterative methods and alternate between two phases, that is, sparse coding and dictionary update. In the sparse coding phase, the sparse representation is updated for a fixed dictionary, while in the dictionary update phase, the dictionary and its corresponding sparse representation are both updated. The MOD algorithm updates all the atoms of the dictionary simultaneously by solving the optimization problem where a matrix inversion operation is used, hence its complexity is high. However, KSVD updates the dictionary atom by atom such that it does not need a matrix inversion operation and is more efficient. ODL is an online stochastic approximation-based optimization algorithm for DL. It can achieve faster performance and better dictionaries than classical batch algorithms for both small and large datasets with lower computational cost. Unlike the above three algorithms, the proposed DL algorithm is a parallel learning approach, which can handle the entire dataset at a time.

In this paper, the authors propose a scheme for the CS of multichannel ECG signals in wireless telehealth system. The proposed scheme compresses the input original multichannel ECG signals and generates the recovered ECG signals from the compressed ECG signals, which is achieved by a reconstruction algorithm and a DL algorithm. The major contributions of this paper are summarized as follows.

- The proposed reconstruction algorithm is designed based on a spatio-temporal sparse model, which can exploit both the temporal correlation of each channel signal and the inter-channel (spatial) correlation among different channel signals. It is capable of directly reconstructing the non-sparse signals.
- The proposed DL algorithm can further enhance the reconstruction performance by exploiting the sparsity of each channel signal in transformed domain.
- Extensive simulation experiments have been implemented to evaluate the performance of the proposed algorithms. Simulation results have demonstrated that the proposed algorithms achieve better performance when compared to the benchmarking schemes.

The rest of the paper is organized as follows. Section 2 introduces the relative models. Section 3 describes the proposed CS scheme. Section 4 presents the experimental results and Sec. 5 concludes the paper.
2. Preliminary

In this paper, the proposed scheme is based on spatio-temporal sparse model, which is an extension of the CS model and used to exploit spatial and temporal correlation of multichannel signals.

2.1. CS model

Compared to the traditional Nyquist sampling theorem which specifies that a signal must be sampled at least two times faster than the signal bandwidth to avoid losing information when capturing the signal, CS is a new method to capture and represent compressible signals at a rate significantly below the Nyquist rate. It measures a small number of nonadaptive linear projections of the signal. The number of these measurements are usually much smaller than that of the samples which define the signal, and the signal is then reconstructed from them. The basic CS model, also called the single measurement vector (SMV) model, is given by

$$y = \Phi x + v,$$

where $x \in \mathbb{R}^{N \times 1}$ is the original signal contaminated by noise and artifacts, $\Phi \in \mathbb{R}^{M \times N}$ is the measurement matrix, $y \in \mathbb{R}^{M \times 1}$ is the compressed signal, and $v$ is a noise vector. The original signal $x$ can be reconstructed by a CS algorithm using the measurement matrix $\Phi$ according to

$$\hat{x} = \arg \min_{x} \| y - \Phi x \|_2^2 + \alpha f(x),$$

where $\alpha$ is a regularization parameter, and $f(x)$ is a penalty function of $x$. The approach is called signal reconstruction in the original domain.

When $x$ is sparse enough, it can be recovered from $y$. However, many signals (e.g., ECG) are not sparse in the original domain, but sparse in some transformed domains. So it needs to seek a dictionary such that $x$ can be sparsely represented as $x = D\theta$, where $D \in \mathbb{R}^{N \times P}$ is called dictionary matrix and $\theta$ is the sparse coefficient vector. In addition, the signal compression in this work is modeled as a noiseless CS problem such that the sensor noise $v$ can be ignored, as in Ref. 34. Note that this does not mean the artifacts and noise in ECG signals are totally ignored, in fact, they are taken into account by the variable $x$. Thus, according to these above reasons, Eq. (1) can be re-written as

$$y = \Phi x = \Phi D\theta = \Theta \theta,$$

where $\Theta = \Phi D$ is the sensing matrix. Then the original signal $x$ can be reconstructed according to

$$\hat{x} = D\hat{\theta} \quad \text{and} \quad \hat{\theta} = \arg \min_{\theta} \| y - \Theta \theta \|_2^2 + \alpha f(\theta).$$

The approach is called signal reconstruction in a transformed domain.
2.2. Spatio-temporal sparse model

A spatio-temporal model is needed to characterize the spatial and temporal correlation. In this paper, a spatio-temporal sparse model proposed in Ref. 34 is used for the CS of multichannel ECG signals. The model is similar to the multiple measurement vector (MMV) model, which is an extension of the SMV model, except that each column of \( X \) has more complicated structures. Compared to the basic CS model, the spatio-temporal sparse model can be applied for multichannel signals that are the subjects to study in this paper. It is expressed as

\[
Y = \Phi X + V,
\]

where \( X \in R^{N \times L} \) is the original multichannel signals, \( \Phi \in R^{M \times N} \) is the measurement matrix, \( Y \in R^{M \times L} \) is the compressed signals, and \( V \in R^{M \times L} \) is a noise matrix, which can be ignored in our application. Similar to Eqs. (2) and (4), \( X \) can be reconstructed by

\[
\hat{X} = \arg \min_X \|Y - \Phi X\|^2_2 + \alpha f(X),
\]

or

\[
\hat{X} = D \hat{\Psi} \quad \text{and} \quad \hat{\Psi} = \arg \min_\Psi \|Y - \Theta \Psi\|^2_2 + \alpha f(\Psi),
\]

where \( \Psi \) is the sparse coefficient matrix.

The \( l \)th columns of the matrix \( X \) and \( Y \), which are denoted by \( X_l \) and \( Y_l \), are the \( l \)th channel of the original signals and the corresponding compressed signals, respectively. It is assumed that \( X \) has the block structure as follows:

\[
X = \begin{bmatrix}
X_{[1]} \\
X_{[2]} \\
\vdots \\
X_{[g]}
\end{bmatrix},
\]

where \( X_{[i]} \in R^{b_i \times L} \) is the \( i \)th block of \( X \), \( \{b_1, b_2, \ldots, b_g\}(\sum_{i=1}^g b_i = N) \) is the block partition, and there are only a few nonzero blocks. The entries in each column of \( X_{[i]} \) have correlation, which is a kind of temporal correlation of a channel signal. Simultaneously, the entries in each row of \( X_{[i]} \) are also correlated, which is called spatial (inter-channel) correlation. The \( i \)th block \( X_{[i]} \), is assumed to obey a Gaussian distribution \( p(\text{vec}(X_{[i]}^T); A, \beta_i, B_i) = N(0, (\beta_i B_i) \otimes A) \) with the unknown parameters \( \{\beta_i, B_i\}_{i=1}^g \) and \( A \), where \( \text{vec}(X_{[i]}^T) \) denotes the vectorization of the matrix \( X_{[i]}^T \) formed by stacking its columns into a single column vector, the nonnegative parameter \( \beta_i \) determines whether \( X_{[i]} \) is a zero block, \( A \in R^{L \times L} \) and \( B_i \in R^{b_i \times b_i} \) are the parameters capturing the spatial and temporal correlation of \( X_{[i]} \), respectively.
Supposing the blocks \( \{ X_1, X_2, \ldots, X_g \} \) are mutually independent, the matrix \( X \) has the distribution
\[
p(\text{vec}(X^T); A, \{ \beta_i, B_i \}_i) = N(0, \Gamma \otimes A), \quad i = 1, 2, \ldots, g,
\]
where \( \Gamma \) is a block diagonal matrix given by \( \Gamma = \text{diag}(\beta_1 B_1, \beta_2 B_2, \ldots, \beta_g B_g) \).

3. Our Approach

The objective of the proposed CS scheme is to compress and reconstruct multichannel ECG signals with DL. The Algorithm 1 called STSMR-DL is developed in this section to present the overview of the proposed CS scheme. Inputs to the algorithm are the original multichannel ECG signals. At first, a sparse binary matrix is generated randomly (Step 1), and is adopted as the measurement matrix \( \Phi \). Then, the original multichannel ECG signals \( X \) are linearly compressed by the measurement matrix using Eq. (5) (Step 2), and the compressed signals \( Y \) are transmitted to a remote terminal via wireless network. Finally, in the remote terminal, the ECG signals are recovered from the compressed signals based on Eq. (7) (Step 3), and the method to obtain the recovered ECG signals is described in Algorithm 2 with \( \Phi \) and \( X \) replaced by \( \Phi D \) and \( \Psi \), where the dictionary matrix \( D \) is trained from Algorithm 3.

The major contributions of this paper are two-fold. On the one hand, the multichannel ECG signals of the proposed scheme are reconstructed from the compressed signals. On the other hand, the reconstruction performance is improved by utilizing a dictionary matrix obtained from the DL method. Unlike the previous work presented by Ref. 19 that only considers temporal correlation of a signal when reconstructing signals, the proposed scheme reconstructs multichannel signals by exploiting spatial and temporal correlation at the same time, and enhances the

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Algorithm 1. The proposed STSMR-DL algorithm for the CS of multichannel ECG signals.

Input: Original multichannel ECG signals \( X \).
Output: Recovered multichannel ECG signals \( \hat{X} \).

Step 1: Generate \( M \times N (M \ll N) \) dimensional sparse binary random measurement matrix \( \Phi \).
Step 2: Compute the \( M \)-dimensional compressed measurement value \( Y \) based on \( \Phi \) and Eq. (5).
Step 3: Based on Eq. (7), obtain the recovered ECG signals \( \hat{X} \) from the compressed signals \( Y \) using Algorithm 2 with \( \Phi \) and \( X \) replaced by \( \Phi D \) and \( \Psi \), where the dictionary matrix \( D \) is trained by Algorithm 3.
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\(^{a}\text{diag}(\beta_1 B_1, \beta_2 B_2, \ldots, \beta_g B_g)\) denotes a diagonal matrix with principal diagonal elements being \( \beta_1 B_1, \beta_2 B_2, \ldots, \beta_g B_g \).
reconstruction performance by using the sparsity of the signals in transformed domain, which is demonstrated in Algorithms 2 and 3. In other words, Algorithm 2 aims at reconstructing multichannel ECG signals by exploiting spatial and temporal correlation, and Algorithm 3 determines a dictionary under which the signals can be sparsely represented using DL method. The details of Algorithms 2 and 3 are presented in the following subsections.

3.1. Reconstruct multichannel ECG signals

The proposed spatio-temporal sparse model based reconstruction algorithm is called STSMR, which is given in Algorithm 2. The algorithm has the ability to directly reconstruct nonsparse signals due to its adoption of sparse Bayesian learning, which can exploit the correlation structures of signals. Inputs to the algorithm are the block partition \( \{b_1, b_2, \ldots, b_g\} \), the compressed multichannel signals \( Y \) and the measurement matrix \( \Phi \) obtained from Algorithm 1. As mentioned before, the multichannel ECG signals \( X \) has the distribution with parameters \( A, \beta_i \) and \( B_i \), as given in Eq. (9). Hence, these parameters need to be first calculated to reconstruct \( X \). In order to reduce the computational load, the spatio-temporal sparse model is divided into two equivalent models, that is, spatial correlated model and temporal correlated model. Then, parameters \( A \) and \( \{\beta_i, B_i\} \) can be derived from the two models, respectively. The proposed algorithm alternates the calculation between the two models until convergence, and the output is the reconstructed multichannel signals \( X \). The details of each step are described in the following subsections.

**Algorithm 2.** The proposed STSMR algorithm to reconstruct multichannel signals.

**Input:** Compressed multichannel signals \( Y \), measurement matrix \( \Phi \), and the block partition \( \{b_1, b_2, \ldots, b_g\} \).

**Output:** Reconstructed multichannel signals \( X \).

**Initialization:** \( B_i \leftarrow I_{b_i} (\forall i), \beta_i \leftarrow 1 (\forall i) \) and \( X \leftarrow \Phi^T \left( \Phi \Phi^T \right)^{-1} Y \)

1: Repeat
2: \( \widehat{A} \leftarrow \sum_{i=1}^g \beta_i^{-1} X_{[i]}^T B_i^{-1} X_{[i]}, \)
3: \( A \leftarrow \frac{\widehat{A}}{\| \widehat{A} \|_F}, \)
4: \( \lambda \leftarrow \Gamma \Phi^T \left( \Phi \Gamma \Phi^T \right)^{-1} Y A^{-\frac{1}{2}}, \)
5: \( \Omega \leftarrow \Gamma - \Gamma \Phi^T \left( \Phi \Gamma \Phi^T \right)^{-1} \Phi \Gamma, \)
6: \( \beta_i \leftarrow \frac{1}{\sum_{i=1}^g} \text{Tr} \left[ B_i^{-1} \left( \Omega_{[i]} + \lambda_{[i]} \beta_i^{-1} \Omega_{[i]}^T \right) \right], \forall i, \)
7: \( B_i \leftarrow \frac{1}{\beta_i} \sum_{i=1}^L \frac{\Omega_{[i]} + \lambda_{[i]} \beta_i^{-1} \Omega_{[i]}^T}{\beta_i}, \forall i, \)
8: \( X \leftarrow \lambda A^\dagger, \)
9: until converge.
3.1.1. Estimate the parameter A

Assume the parameters $\beta_i$, $B_i$, and $X$ are known, which are initialized to 1, $I_{b_i}$ and $\Phi^T(\Phi\Phi^T)^{-1}Y$, respectively. Let $\hat{\Phi} = \Phi B^\dagger$, $\hat{X} = B^{-\frac{1}{2}} X$, where the matrix $B$ is defined as $B = \text{diag}(B_1, B_2, \ldots, B_y)$,\(^b\) the original model (Eq. (5)) becomes

$$Y = \hat{\Phi} \hat{X},$$

where $\hat{X}$ has the same block structure as $X$, but there is no temporal correlation in its each block. Clearly, Eq. (10) which is called spatial correlated model is similar to the MMV model in Ref. 36 but with block structures in each column of $\hat{X}$. Thus, following the method used to derive the T-MSBL algorithm,\(^3\) the parameter $A$ can be estimated as $A \leftarrow \hat{A}/\|\hat{A}\|_F$ with $\hat{A} \leftarrow \sum_{i=1}^{g} \beta^{-1}_i X^T_{[i]} B_i^{-1} X_{[i]}$. (lines 2–3). It has been assumed that the parameters $\beta_i$ and $B_i$ are given in this model. Such parameters can be calculated in the temporal correlated model described below.

3.1.2. Estimate the parameters $\beta_i$ and $B_i$

Let $\hat{Y} = YA^{-\frac{1}{2}}$ and $\hat{X} =XA^{-\frac{1}{2}}$, where the parameter $A$ is estimated from spatial correlated model, then Eq. (5) becomes

$$\hat{Y} = \Phi \hat{X},$$

where the columns of $\hat{X}$ are independent. Therefore, the spatial correlation in them is eliminated. Obviously, Eq. (11) which is called temporal correlated model is a extension of the block sparse MMV model exploiting intra-block correlation.\(^1\) Hence, the parameters $\beta_i$, $B_i$ and $X$ can be estimated using the expectation maximization approach in Ref. 19, as is shown in lines 4–8 of Algorithm 2. It operates as follows. First, the prior for $\hat{X}$ is given by $p(\hat{X}; \Gamma) = \prod_{i=1}^L p(\hat{X}_j) \sim \prod_{i=1}^L N(0, \Gamma)$, where $\hat{X}_j$ is the $i$th column of $\hat{X}$. Then, the likelihood is $p(\hat{Y}_j|\hat{X}_j) = N(\Phi \hat{Y}_j, 0)$. Hence, the posterior can be expressed as $p(\hat{X}_j|\hat{Y}_j; \Gamma) = N(\lambda_j, \Omega)$ with the mean (line 4) $\lambda_j \leftarrow \hat{\Gamma} \Phi^T (\Phi \Phi^T)^{-1} \Phi^T \hat{\Gamma}$ and the covariance matrix (line 5) $\Gamma \leftarrow \Gamma - \Gamma \Phi^T (\Phi \Phi^T)^{-1} \Phi$. The parameters $\beta_i$ and $B_i$ (vi) (lines 6–7) are estimated using the expectation maximization approach\(^3\) that is an iterative method for finding maximum likelihood or maximum a posteriori (MAP) estimates of parameters in models, as given by $\beta_i \leftarrow \frac{1}{L^T} \sum_{l=1}^L \text{Tr}[B_i^{-1}(\Omega_{[i]} + \lambda_{i|i} \lambda_{[i]}^T)]$ and $B_i \leftarrow \frac{1}{L} \sum_{l=1}^L \frac{\Omega_{[i]} + \lambda_{i|i} \lambda_{[i]}^T}{\beta_i}$, respectively. So the MAP estimate of $\hat{X}$ is given by $\hat{X} \leftarrow \lambda = \hat{\Phi} \Phi^T (\Phi \Phi^T)^{-1} \hat{Y}$, and the reconstructed multichannel signals $X$ (line 8) can be obtained as $X \leftarrow \hat{X} A^\dagger = \lambda A^\dagger$.

\(^b\)diag$(B_1, B_2, \ldots, B_y)$ denotes a diagonal matrix with principal diagonal elements being $B_1, B_2, \ldots, B_y$. 

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3.2. Learn dictionary

As opposed to selecting a pre-existing basis (such as Fourier and Wavelet), DL or training is a more recent approach to dictionary design by using a signal or a set of signals, which has been shown to better capture the structure and features specific to the signals being analyzed. The dictionary is a transform domain where the signal can be represented with only a few coefficients. It has been observed that learning a good dictionary can substantially improve the reconstruction performance.\textsuperscript{22–25} In this section, a DL algorithm, which is used to enhance the reconstruction performance of the STSMR algorithm by exploiting the sparsity of each channel signal in transformed domain, is proposed. The basic idea behind the algorithm is to learn local dictionaries for each channel and merge them to form a global dictionary.

Given a set of $J$ training vectors of multichannel signals $\{\tilde{X}_i\}_{i=1}^J \in \mathbb{R}^{N \times L}$, where $L$ is the number of channels and $N$ is the signal length. The objective is to learn a dictionary $D \in \mathbb{R}^{N \times P}$ where $P$ is the number of dictionary atoms, such that $D$ represents each $\tilde{X}_i$ with a $k$-sparse coefficient matrix $\Psi_i$, that is, $\|\tilde{X}_i - D\Psi_i\|_2 \leq \varepsilon$, where $\Psi_i$ satisfies $\|\Psi_i\|_0 \leq k \ll P, \forall i$. Let the matrix $\tilde{X} \in \mathbb{R}^{N \times (L \times J)}$ denote the whole training dataset formed by stacking $\tilde{X}_i$ on the columns. The proposed DL approach given in Algorithm 3 operates as the following four steps.

At the first step, the training datasets $\tilde{X}^l (l = 1, 2, \ldots, L)$ of $L$ channels are formed from the whole training dataset $\tilde{X}$. At the second step, a dictionary which is referred to as local dictionary is learned from each $\tilde{X}^l$ with a sparsity level of $k_1 < k$, where $k$ is the desired sparsity level for the whole dataset, and let the local dictionary of the $l$th channel signal be denoted by $D^l \in \mathbb{R}^{N \times P_l} (P_l \leq P, l = 1, 2, \ldots, L)$. At the third step, a new dataset is constructed by stacking the local dictionaries $D^l$ on the columns. At the last step, a global dictionary $D$ that can represent the whole dataset with a sparse coefficient matrix is trained on this new dataset with sparsity $k_2$. The values of $k_1$ and $k_2$ are set to satisfy $k = k_1 k_2$.

Algorithm 3. The proposed DL algorithm.

**Input:** Training dataset $\tilde{X} \in \mathbb{R}^{N \times (L \times J)}$ and sparsity level $k$.

**Output:** Dictionary matrix $D \in \mathbb{R}^{N \times P}$.

**Step 1:** Form the training datasets $\tilde{X}^l (l = 1, 2, \ldots, L)$ of $L$ channels from the whole training dataset $\tilde{X}$.

**Step 2:** Learn a local dictionary $D^l \in \mathbb{R}^{N \times P_l} (P_l \leq P, l = 1, 2, \ldots, L)$ from each $\tilde{X}^l$ with an $k_1$-sparse ($k_1 < k$) coefficient matrix $\Psi^l$.

**Step 3:** Form a new dataset $W = [\tilde{D}^1, \tilde{D}^2, \ldots, \tilde{D}^L] \in \mathbb{R}^{N \times P_1 L}$, where $\tilde{D}^l = \|\Psi^l\|_F D^l$.

**Step 4:** Train a global dictionary $D$ on the new dataset $W$ with sparsity $k_2 = k_k_1$.
4. Experiments and Results

In this section, extensive simulation experiments are carried out to evaluate the performance of the proposed algorithms.

4.1. Experimental settings

In our experiments, two benchmarking approaches are chosen to compare with the proposed STSMR algorithm. The first benchmarking scheme, referred to as NO-STSMR, does not exploit spatial and temporal correlation by fixing the parameters $A$ and $B_j (\forall i)$ to identity matrices in STSMR. It is utilized as a baseline to show the highest efficiency of the proposed STSMR algorithm in improving the reconstruction quality. The second benchmarking scheme, referred to as BSBL-BO,\textsuperscript{19} exploits the temporal correlation of the signals and recovers the signals channel by channel when the number of channels is large. It is a state-of-the-art scheme and is compared to demonstrate the effectiveness of the proposed STSMR algorithm. In addition, we also compare the proposed two algorithms, STSMR and STSMR-DL, to show that DL could further improve the reconstruction quality.

The dataset signal01 in the open source electrophysiological toolbox (OSET),\textsuperscript{37} which contains eight-channel real fetal cardiac signals, is used in the simulation. More specifically, the first 6,400 time points of each down-sampled recording are selected as the testing dataset for reconstruction algorithm to construct the testing signals $X$, and the rest are selected as the training dataset for DL algorithm to construct the training signals $\tilde{X}$. For convenience, the testing dataset is divided into 50 epochs and each epoch consists of 128 time points. To compress the signals epoch by epoch, a $M \times N$ sparse binary matrix is used as the measurement matrix $\Phi$. The value of $N$ is fixed to 128 while the value of $M$ is variable. Then, the compression ratio (CR) that is given by $CR = \frac{N \times M}{N} \times 100\%$, as defined in Refs. 9, 21, 38 and 39, can be varied from 20\% to 90\%. For each value of $M$, the experiment repeats 25 times and the matrix $\Phi$ is randomly generated in each time. In matrix $\Phi$, 12 elements are randomly selected from each column and set to 1 while other elements are set to 0. The block partition is $b_1 = b_2 = \cdots = b_{16} = 16$ and the maximum number of iterations is set to 30.

To quantify the performance of the proposed scheme, three performance metrics are used in the simulation. The first performance metric is the percentage root-mean-square difference (PRD), defined as $PRD = \frac{\|x - \tilde{x}\|_2}{\|x\|_2} \times 100\%$, where $x$ and $\tilde{x}$ are the original and the reconstructed signals, respectively. Obviously, the smaller the PRD is, the better the reconstruction performance could be. The second performance metric, is the Pearson correlation between the extracted clean fetal ECG from original signals and the extracted one from reconstructed signals by utilizing the fast independent component analysis (ICA) algorithm.\textsuperscript{40} The ICA algorithm is a statistical method for transforming an observed multidimensional random vector into
components that are statistically as independent from each other as possible. It is developed based on an optimization iteration scheme designed to maximize non-Gaussianity to achieve statistical independent components (ICs). Pearson correlation was proposed in Ref. 9 to better detect small recovery errors for structured signals. The value of Pearson correlation is in the range of [0,1]. Clearly, the larger the Pearson correlation is, the higher the reconstruction quality could be. The first two performance metrics are employed to evaluate the effectiveness of the proposed scheme in improving reconstruction quality. Then, the third performance metric which is used to evaluate the proposed scheme with respect to time overhead, is the CPU time taken by the algorithm. All the algorithms are implemented in MATLAB version 7.11.0 (R2010b), and the simulation is performed on a computer with 3.0 GHz CPU and 2 GB RAM.

4.2. Experimental results

In the simulation, we first compare the reconstruction performance of the proposed algorithms with that of benchmarking algorithms using the performance metrics PRD and Pearson correlation. We then compare the average CPU time taken by the proposed algorithms with that of benchmarking algorithms to evaluate the timing overhead. Moreover, we derive the recovered ECG recordings and extracted clean fetal ECG recordings to visually show the effectiveness of the proposed algorithms.

4.2.1. Evaluate the reconstruction performance using metric PRD

The average PRDs (%) of eight channels using four algorithms NO-STSMR, BSBL-BO, STSMR and STSMR-DL under varying CRs (CR = 20%, 30%, 40%, 50%, 60%, 70%, 75%, 80%, 85% and 90%) are listed in Table 1. As can been seen from the table, the average PRDs of the proposed STSMR algorithm for each channel is much smaller than that of methods NO-STSMR and BSBL-BO. For example, in the case of CR = 50%, the average PRDs of eight channels using NO-STSMR, BSBL-BO, and STSMR are 29.36%, 5.39% and 4.20%, respectively; in the case of CR = 70%, the average PRDs of eight channels using NO-STSMR, BSBL-BO, and STSMR are 29.36%, 5.39% and 4.20%, respectively; in the case of CR = 70%, the average PRDs of eight channels using NO-STSMR, BSBL-BO, and STSMR are 30.23%, 18.15% and 12.69%, respectively. Even when the CR is high (CR = 90%), the STSMR algorithm can still achieve a lower PRD than that of NO-STSMR and BSBL-BO. That is, the average PRDs of eight channels achieved by STSMR is 5.14% lower than that of NO-STSMR and 2.83% lower than that of BSBL-BO. According to the observation that the smaller the PRD is, the better the reconstruction performance could be, we can induce that the proposed algorithm STSMR outperforms the NO-STSMR and BSBL-BO. In addition, Table 1 also shows the average PRDs of the STSMR-DL algorithm. It has been demonstrated in the table that the average PRDs of STSMR-DL is smaller than that of STSMR. For instance, in the cases of CR = 70% and 90%, the average PRDs of eight channels achieved by STSMR-DL are
Table 1. The average PRDs (%) of eight channels using four algorithms NO-STSMR, BSBL-BO, STSMR and STSMR-DL under varying CRs (CR = 20%, 30%, 40%, 50%, 60%, 70%, 75%, 80%, 85% and 90%).

| Channel 1 | 23.04 | 3.58 | 2.22 | 2.07 | 29.55 | 4.89 | 3.34 | 2.98 |
| Channel 2 | 14.73 | 2.05 | 1.47 | 1.47 | 18.45 | 2.75 | 2.03 | 2.01 |
| Channel 3 | 15.71 | 2.57 | 1.52 | 1.36 | 20.86 | 3.53 | 2.14 | 1.87 |
| Channel 4 | 12.36 | 1.51 | 1.30 | 1.29 | 16.70 | 2.02 | 1.75 | 1.81 |
| Channel 5 | 18.45 | 2.67 | 1.79 | 1.69 | 24.33 | 3.67 | 2.47 | 2.34 |
| Channel 6 | 21.55 | 2.20 | 1.36 | 1.30 | 26.69 | 3.03 | 2.00 | 1.77 |
| Channel 7 | 12.24 | 1.89 | 1.48 | 1.53 | 16.07 | 2.44 | 2.08 | 2.15 |
| Channel 8 | 12.74 | 1.76 | 1.27 | 1.29 | 17.25 | 2.36 | 1.82 | 1.85 |
| Average  | 16.35 | 2.28 | 1.55 | 1.50 | 21.24 | 3.09 | 2.20 | 2.10 |

| Channel 1 | 33.23 | 6.10 | 4.40 | 3.84 | 44.58 | 8.78 | 6.71 | 5.34 |
| Channel 2 | 19.11 | 3.35 | 2.61 | 2.57 | 24.38 | 4.65 | 3.73 | 3.50 |
| Channel 3 | 22.63 | 4.61 | 2.93 | 2.46 | 30.90 | 6.51 | 4.38 | 3.36 |
| Channel 4 | 16.13 | 2.37 | 2.21 | 2.33 | 19.60 | 3.14 | 2.94 | 3.08 |
| Channel 5 | 27.08 | 4.44 | 3.30 | 3.18 | 34.38 | 6.21 | 4.66 | 4.46 |
| Channel 6 | 30.24 | 3.90 | 2.81 | 2.44 | 36.94 | 5.62 | 4.12 | 3.48 |
| Channel 7 | 16.96 | 2.93 | 2.71 | 2.77 | 21.35 | 4.06 | 3.60 | 3.75 |
| Channel 8 | 17.56 | 2.90 | 2.47 | 2.43 | 22.80 | 4.13 | 3.43 | 3.42 |
| Average  | 22.87 | 3.82 | 2.93 | 2.75 | 29.36 | 5.39 | 4.20 | 3.80 |

<p>| Channel 1 | 41.49 | 13.20 | 10.92 | 7.25 | 48.82 | 25.90 | 21.64 | 12.92 |
| Channel 2 | 25.98 | 6.65 | 5.82 | 4.79 | 25.39 | 17.49 | 10.60 | 7.37 |
| Channel 3 | 31.01 | 9.60 | 7.55 | 4.33 | 32.73 | 17.80 | 15.18 | 7.85 |
| Channel 4 | 22.62 | 4.03 | 4.09 | 3.90 | 16.98 | 13.73 | 6.38 | 5.27 |
| Channel 5 | 34.31 | 9.13 | 7.57 | 5.74 | 34.72 | 19.97 | 14.47 | 9.87 |
| Channel 6 | 34.84 | 8.63 | 7.04 | 4.89 | 38.44 | 20.65 | 14.14 | 8.52 |
| Channel 7 | 23.04 | 5.57 | 5.15 | 4.80 | 21.62 | 14.71 | 9.13 | 6.74 |
| Channel 8 | 24.13 | 6.04 | 5.23 | 4.28 | 23.16 | 14.96 | 10.00 | 6.35 |
| Average  | 29.68 | 7.85 | 6.67 | 5.00 | 30.23 | 18.15 | 12.69 | 8.11 |</p>
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4.58% and 6.79% smaller than that of STSMR, respectively. This indicates that the STSMR-DL algorithm can further improve the reconstruction quality as compared to the STSMR via the DL technique adopted.

4.2.2. Evaluate the reconstruction performance using metric pearson correlation

Figure 1 plots the average Pearson correlation of four algorithms under varying CRs (CR = 20%, 30%, 40%, 50%, 60%, 65%, 70%, 75%, 80%, 85% and 90%). As expected, the results in the figure show that the proposed algorithm STSMR can achieve a larger Pearson correlation as compared to the baseline method NO-STSMR and the state-of-the-art method BSBL-BO for the 11 CRs under test. To be specific, when the CRs are relatively low (CR = 20%, 30%, 40%, 50%, 60%, 65% and 70%), both STSMR and BSBL-BO show good results of Pearson correlation, which are much higher than that of NO-STSMR. For example, in the case of CR = 65%, the Pearson correlation achieved by STSMR, BSBL-BO and NO-STSMR are 0.90, 0.81 and 0.16, respectively. When the CRs are relatively high (CR = 75%, 80%, 85% and 90%), the Pearson correlation achieved by STSMR and BSBL-BO drops a lot, but is still higher than that of NO-STSMR. For instance, in the case of CR = 85%, the Pearson correlation achieved by STSMR, BSBL-BO and NO-STSMR is 0.42, 0.18 and 0.1, respectively. Furthermore, as demonstrated in the two examples, it is also easy to find that the proposed algorithm STSMR has a higher Pearson correlation as compared to the state-of-the-art method BSBL-BO, especially when the CR is high.

![Fig. 1. Average Pearson correlation of four algorithms NO-STSMR, BSBL-BO, STSMR and STSMR-DL under varying CRs (CR = 20%, 30%, 40%, 50%, 60%, 65%, 70%, 75%, 80%, 85% and 90%) (color online).](image-url)
According to the observation that the larger the Pearson correlation is, the higher the reconstruction quality could be, we can conclude that the proposed algorithm STSMR outperforms the NO-STSMR and BSBL-BO methods. This benefits from the exploitation of inter-channel correlation among different channel signals in the algorithm STSMR. However, similar to benchmarking algorithms BSBL-BO and NO-STSMR, our algorithm STSMR inevitably becomes less effective in reconstruction when the CR is high. This is because the derived measurements under high CR cannot preserve the structure of original signals, which undermines the foundation of our method. Figure 1 also shows that STSMR-DL has a larger Pearson correlation than STSMR and hence achieves a higher recovery performance. This is due to the utilization of dictionary matrix obtained from the DL algorithm adopted in the STSMR-DL.

4.2.3. Compare the CPU time taken by four algorithms

The average CPU time taken by every algorithm to reconstruct the eight-channel recordings under varying CRs is plotted in Fig. 2. As can be seen from the figure, the CPU time consumed by the algorithm STSMR is much less than that of BSBL-BO and is similar to that of NO-STSMR. The reason is that STSMR can jointly recover the multichannel signals simultaneously while BSBL-BO cannot and has to recover the signals channel by channel. In the meantime, it has been shown in the figure that STSMR-DL, which reconstructs the signals with DL technique, is slower than STSMR, which directly reconstructs the signals without using any dictionary. This is because STSMR-DL has to take extra time to train dictionary for achieving better reconstruction quality.

![Fig. 2. Average CPU time taken by four algorithms NO-STSMR, BSBL-BO, STSMR and STSMR-DL under varying CRs (CR = 20%, 30%, 40%, 50%, 60%, 65%, 70%, 75%, 80%, 85% and 90%) (color online).](image-url)
4.2.4. Compare the signal recovery quality using visual recordings

To visually demonstrate the signal recovery quality, we first derive the original recordings and recovered recordings for the four algorithms when CR = 70%, as illustrated in Fig. 3. Clearly, both of the benchmarking approaches NO-STSMR and BSBL-BO cannot recover the signals with satisfactory quality at such few measurements. This is because NO-STSMR does not exploit spatial and temporal correlation, and BSBL-BO cannot jointly recover the multichannel signals. In contrast, even if the signals are compressed by 70%, the recovered signals by the proposed algorithms are still of good quality, due to the advantage of exploiting the correlation

![Fig. 3. The original recordings and reconstructed recordings by four algorithms NO-STSMR, BSBL-BO, STSMR and STSMR-DL (CR = 70%) (color online).](image-url)
among signals from different channels. Especially, the distortion of the results of STSMR-DL is smaller.

We then extract the clean fetal ECG from the original signals using the FastICA algorithm and compare it with the one extracted from the reconstructed signals. Specifically, FastICA is first performed on the original recordings given in Fig. 3(a), and five ICs can be extracted, as shown in Fig. 4(a). Note that, among the five ICs, the fourth IC is the extracted clean fetal ECG signal. FastICA is then performed on

![Fig. 4. ICs of original recordings and reconstructed recordings by four algorithms NO-STSMR, BSBL-BO, STSMR and STSMR-DL (CR = 70%). (The fourth IC is the extracted clean fetal ECG signal).](image)

(e) Reconstructed recordings by STSMR-DL.

Fig. 3. (Continued)
the recovered recordings (i.e., Figs. 3(b) and 3(c)) of the benchmarking methods NO-STSMR and BSBL-BO when CR = 70%. The extracted ICs are shown in Figs. 4(b) and 4(c), respectively. It is clear that the two ICs are significantly distorted, and the clean fetal ECG is not extracted. Next, the same procedure is performed on the recovered recordings (i.e., Figs. 3(d) and 3(e)) achieved by the proposed algorithms STSMR and STSMR-DL. The generated ICs are depicted in Figs. 4(d) and 4(e). Obviously, the difference is very small, indicating that the reconstruction quality of the proposed algorithms are satisfactory, or even better when the ICs of STSMR-DL almost exactly match the ICs of the original recordings.
5. Conclusion

In this paper, a spatio-temporal sparse model-based reconstruction algorithm for the recovery of multichannel ECG signals, namely, the STSMR algorithm, is proposed. It exploits both temporal correlation in a signal itself and spatial correlation among signals from different channels. Furthermore, a DL algorithm is proposed. It yields a dictionary which can be used combined with the STSMR algorithm. Extensive simulation results demonstrate that the STSMR algorithm has a better reconstruction performance for multichannel ECG signals as compared to the benchmarking methods. Also, by using a dictionary, which is obtained utilizing the proposed DL algorithm, the reconstruction performance of the STSMR algorithm can be greatly improved.

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